Analysis of Imprecise Perception in Route Choice Considering Fuzzy Costs

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Abstract. In this work we describe a path choice model in which costs are represented by fuzzy numbers in order to consider uncertainty of users and trying to overcome some limits of well know logit based choice models. Users' estimation costs in order to choice a route over a road network is made in a real uncertain, imprecise and ambiguous environment. As suggested by research in this field, fuzzy numbers is a suitable framework to model such choice context. In this paper, we analyse path costs represented by Fuzzy Numbers. For the fuzzification of the crisp value of the cost we introduce a parameter, that aims at representing the imprecision of users in path cost perception. We assume that users associate a preference index to the paths according to the estimated costs and that the probability of choice of a path is proportional to the preference index. Moreover, we introduce an index for evaluating the overlapping degree of each path with the others. In this way, we overcome the problem due to the Independence from Irrelevant Alternatives (IIA) of Logit based models, which rises for overlapping paths. The method allows to reproduce user choices as compared to other well established ones.

Keywords: Traffic assignment · Route choice · Fuzzy sets · Uncertainty · Overlapping paths

1 Introduction

Traffic assignment models (TAMs) are powerful tools to reproduce users' choices over a transport network. TAMs are based on path choice models; traditional path choice models (deterministic models and stochastic models) are based on the comparison between path costs with crisp numbers. The deterministic choice models assumes that users perceive exactly each path cost, and choose the one having the maximum utility. Thus, each user chooses minimum cost path, and does not consider any other path. This leads to the well-known "All or Nothing" assignment model.

Generally, reproduction of real perceived cost is not an easy task, and the analyst does not exactly know the utility that users associate to each alternative (perceived utility). The *perceived utility* can be different from the average value (i.e. *systematic utility*) [23]. Therefore, some users can choose a path different from the minimum cost

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one, and consequently, a choice probability greater than zero occurs for all the possible paths belonging to the users' choice set.

To reproduce this situation the well-established *random utility models* are used (Logit, Gammit, Probit and their further specifications are the most known) (see for ex. [7]).

Several models based on fuzzy set theory for network assignment have been proposed in literature [2, 10, 11, 20, 24–26].

Fuzzy set theory-based route choice model was proposed for the first time by Teodorović and Kikuchi [27]. Their research was followed by many works using fuzzy set theory and based on different approaches.

Wang and Liao [28] considered the traffic assignment problem when the elements of the incidence matrix representing the graph of the transportation network is assumed fuzzy, in the sense that, the arcs chosen into the desired path for travelling from and Origin to a destination is represented as uncertain. Lotan and Koutsopoulos [16] developed a model that uses the concepts of approximate reasoning for route choice in the presence of information by mean of fuzzy inference approach. Peeta and Yu [21] set up a hybrid (fuzzy–probabilistic) model to study the effect of information on travellers' behaviour. Chang and Chen [9] formulated a link-based fuzzy user-optimal route choice problem embedding link interactions using the variational inequality approach.

Henn [14] proposed a route choice model taking account of the imprecision and the uncertainty lying in the dynamic choice process. Okada and Soper [19] focused their work on a shortest path problem on a network in which a fuzzy number is used to represent the length of each arch belonging to the network. Henn and Ottomanelli [15] analysed the role of uncertainty based on the Possibility Theory framework and discussed the pitfalls of some random utility route choice models. They focused on two types of uncertainty: route capacity and free-flow costs. The proposed model, based on possibility theory, shows the different choice behaviour with respect the two uncertainties. Bin et al. [3] proposed a traffic assignment model that is solved by a heuristic-based algorithm which combines the fuzzy shortest path algorithm and the C-Logit method.

Ghatee and Hashemi [12] assumed a fuzzy level of travel demand and found a fuzzy equilibrium flow that satisfies a quasi-logit formula for the network. Murat and Uludag [18] presented route choice model of transportation network based on fuzzy logic model (FLM), logistic regression model (LRM) and survey-based data. The model considered four parameters (traffic safety, travel time, congestion and environmental effects).

Recently, in [17] a method for fusing data relevant both to drivers' experience and provided information about travel time is proposed. The method was based on the Uncertainty-based Information Theory and takes into account the "compatibility" of data originating from different sources, and provides information about acceptability of results.

Our work starts from the work by Akiyama and Kawahara [1] that modelled the route choice behaviour with the values of possibility between fuzzy goal and the fuzzy travel time for the individual routes.

The purpose of this paper is to define a simplified route choice model able to reproduce the imprecision, uncertainty and vagueness typical of the perceived costs. In our hypothesis the user will choice the path according to the comparison between the estimated costs for all the paths. These values are imprecise, therefore the user must choose on the base of ambiguous quantities and on the base of approximate computing. Such an environment can be represented and handled in the framework of fuzzy numbers. The effective travel time will respect the foresight *in a certain measure*, due to the fuzzy nature of the problem. This measure decreases as much as the effective cost is far from the estimated value.

The paper is developed as it follows. In Sect. 2, we describe in detail the path choice model and some examples on simplified cases. In Sect. 3, we describe the results on a test network with the equilibrium assignment. Finally, in the last section, we focus on the conclusions and possible developments of the research.

2 The Route Choice Model

The proposed model is based on the following basic assumptions:

1. The user "*i*" considers all the alternatives, which define his *choice set*I^i.
2. The user "*i*" associates to each alternative $j \in I^i$ the perceived utility U_j^i
3. The utility U_j^i assigned to each alternative depends on a number of measurable characteristics X_j^i, or *attributes,* that are related to the alternative itself and to the user; then $U_j^i = U^i\left(X_j^i\right)$.
4. The utility that the user "*i*" associates to alternative *j* is not exactly known by the analyst, and in every case it is imprecise for the users. Such perceived utility can be represented by a fuzzy number.

On the basis of the given assumptions, it is possible to define the probability that the decision-maker (i.e. user) will select the alternative *j* conditional on his choice set I^i proportionally to the preference associated to this alternative, as it will be shown further on.

2.1 The Representation of Path Costs by Fuzzy Number

Path and arc costs are specified by means of Triangular Fuzzy Numbers (TFN) that is represented by the triple (x_L, x, x_R), where x_L, x and x_R are, respectively, the lower (or left) limit, the central value and the upper (or right) limit. The difference $|x_R - x_L|$ is the so-called "support" of the TFN. To obtain the fuzzy number relevant to the crisp value "x" (i.e. fuzzification) [4] we use a parameter $\alpha \in [0, 1]$ and then we determine the relevant TFN by computing the characteristics points given by the lower (x_L) and the upper (x_R) limits of the TFN as follows:

$$x_R = x \cdot (1 + \alpha); \; x_L = x \cdot (1 - \alpha)$$

In Fig. 1 we represent the TFN corresponding to the values 5 and 7, considering $\alpha = 0.200$.

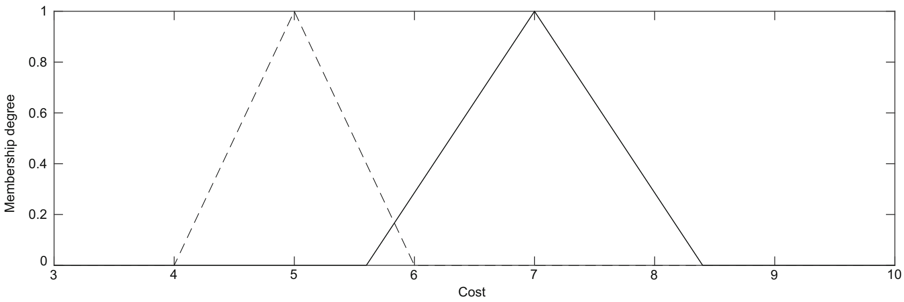

Fig. 1. Representation of costs by Triangular Fuzzy Numbers

Let us consider the sum x + y = z, with x and y *crisp* values. If we add the correspondent TFN (x, y) we obtain "x⊕y = z". In other words, if the value of α is the same for all the TFNs, then the TFN obtained as result of crisp sum is equal to the sum of the TFNs relative to the crisp terms. In this way, our method respects the *independence by the segmentation of the arcs*.

Let consider the following example: let assume two independent paths, the first one made up of a single arc whose costs is 30 and a second one constituted by five arcs whose costs is 6. By applying the proposed method, the two paths have equal crisp cost and equal fuzzy cost: the correspondent TFNs (that represent two paths with the same utility) are identical, as it must be.

The parameter α represents the imprecision of users' prediction (or perception) of path travel time/cost. If this imprecision is null, we assume α = zero. In this case the choice is based on crisp values: the minimum path(s) cost only is (are) chosen. If users will perceive as convenient also other paths (not only the minimum cost one), they will split on them: in this case $\alpha \neq 0$. The value of α is the same for all the paths (only if we have ATIS systems on the network we could have more values for α). Therefore, the support (i.e. base) of the TFN corresponding to the cost "X" (equal to 2αX) is variable, and depends on the crisp value of the cost.

Each user has a different ability to predict (assess) the path travel time. We cannot define a value for each user. Thus, for sake of simplicity, it is advisable to consider an average value of α as representative for all the users belonging to the same class (or category).

Let us analyze the dependence of α on Origin-Destination (O/D) pair. We assume that α is the same for all O/D pairs, i.e. users perceive the state of the network in the same way. If we assign different values of α to different O/D pairs, we would assume that the users of each O/D pair perceive differently the network state. Moreover, we assume that the user is not able to predict the travel time on a certain arc more accurately than other arcs[1]. Thus, we suggest that the value of α will depend from the *urban network*, the *user class* and the *time period*. Naturally, the parameter α have to be

[1] This assumption may fall if in the network there are some VMS. In this situation it might be realistic reduce the α value for the relative arc. In this study, we assume that in the network there aren't VMS.

calibrated. Notwithstanding, we will not discuss about calibration of α since it is out of the scope of this paper. On this topic, the interested reader could refers to the works by Cantarella and Fedele [5, 6], Quattrone and Vitetta [22] or by Hawas [13].

2.2 Definition of the Choice SetI[i]

We assume that the fraction of the demand d_{od} on each path is proportional to its convenience. The proposed method is based on the research of "non dominated[2]" paths for the O/D pair. We carry on this research by the comparison of the characteristic points of every TFN. We find the non-dominated paths set by the *dominance check*. A path is considered as non-dominated from the minimum cost one if the lower limit of its TFN is lower than the upper limit of the TFN of the minimum cost path. In Fig. 2 the dashed TFN (20, 25, 30) represent the fuzzy minimum cost path, while the continuous line represents a non-dominated path since its TFN's lower limits (i.e. $x_L = 24$) is lower that the upper limit of the minimum cost TFN (i.e. $x_R = 30$).

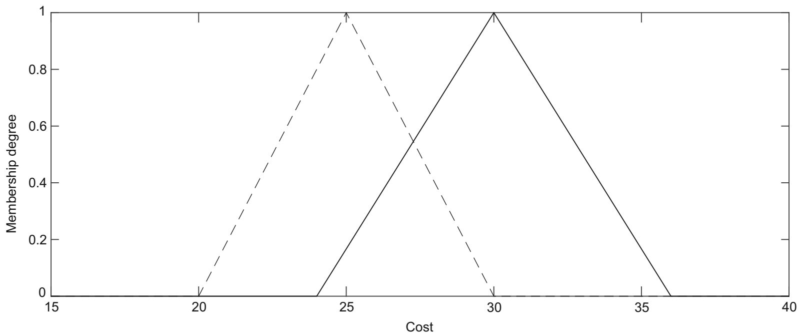

Fig. 2. Example of non-dominated paths

To obtain the non-dominated paths set for each O/D pair without explicit paths enumeration it is possible to use the algorithm proposed in [19] based on the multiple labelling approach, where a number of non-dominated paths can be generated. In addition, the multiple labelling approaches are an exhaustive approach, in which all the possible paths from the source node to the other nodes have to be compared. To this purpose, we modify the algorithm in the step of dominance check according to our method.

The value of α plays an important role in the phase of selection of the paths that will define the choice set. The number of the paths in the choice set depends from the imprecision of costs estimation: the more imprecise are the cost, the higher is the number of elements in the choice set.

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[2] The "non dominated" paths are the most convenient paths [19].

For example, let us consider three paths (A, B, C). Let their estimated costs be equal respectively to x≅25, y≅30 and z≅40 min. The relevant TFN can be represented by the following triples:

$TFN_A(x_L, 25, x_R)$, $TFN_B(y_L, 30, y_R)$ and $TFN_B(z_L, 40, z_R)$ where

$$x_R = x \cdot (1 + \alpha);\; x_L = x \cdot (1 - \alpha)$$

$$y_R = y \cdot (1 + \alpha);\; y_L = y \cdot (1 - \alpha)$$

$$z_R = z \cdot (1 + \alpha);\; z_L = z \cdot (1 - \alpha)$$

If we aim at modelling low imprecision, α must assume a low value. Let α be equal to 0.050. We see that paths B and C are dominated from A, as a result only minimum cost path is considered by users in their choice set (Fig. 3).

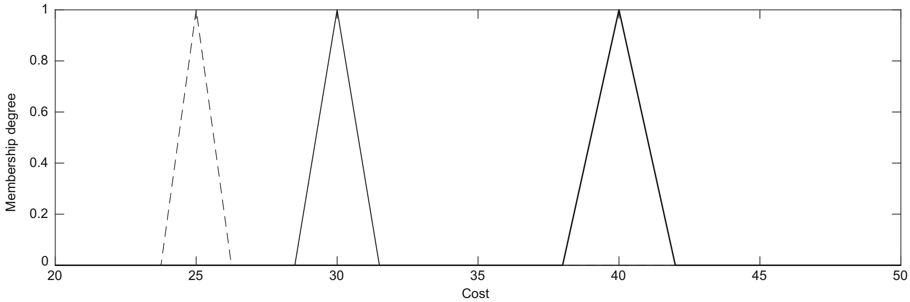

Fig. 3. Example of dominated paths (α = 0.050)

If we want model more imprecision in perceived cost than the previous case, α must assume a higher value. Let α be equal to 0.150 (Fig. 4). In this case the path C only is dominated by A: the choice set is made up of paths A and B.

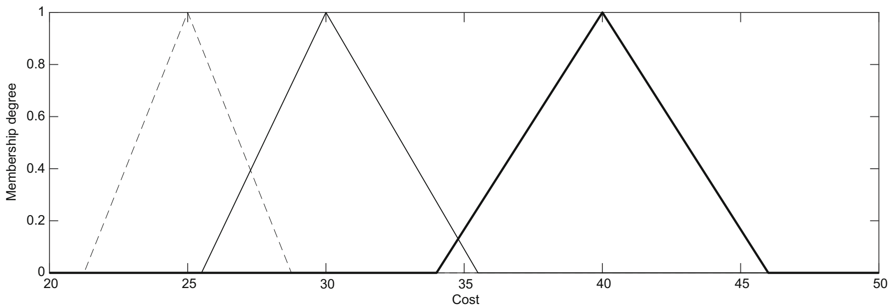

Fig. 4. Example of dominated paths with α = 0.150

As final case, let be $\alpha = 0.300$. In this case there are not dominated paths, so that the choice set is made up of the paths A, B, C all (Fig. 5).

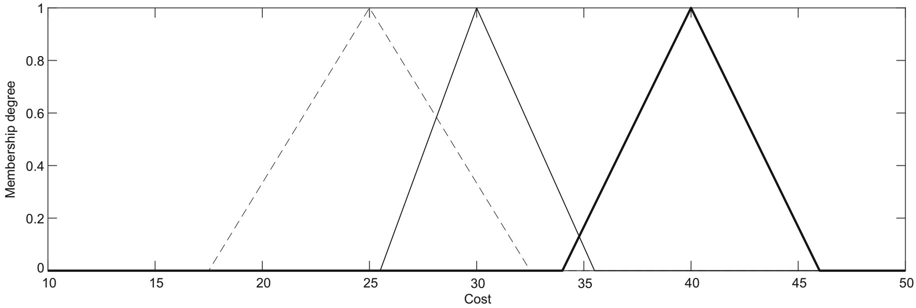

Fig. 5. Example of dominated paths with $\alpha = 0.300$

2.3 Coefficients of Preference

In the previous section, we have shown how to determine (by the dominance check) the choice set for every O/D pair. In this section we are going to show how the demand d_{od} are split among the paths. We assume that the share of demand that will load each path be proportional to the path convenience (utility). In our hypothesis users associate a preference index to each path according to the estimated (perceived) costs.

In this phase, consider the case of independent paths (i.e. they do not have any shared sub-path). In the next section it will be shown how to deal with the case of shared (overlapping) sub-paths.

Then, we associate a numerical value to the convenience of each non-dominated paths. We define for the i-th path the coefficient of preference as follows:

$$C_{pref,i} = (\text{min-cost}, R - i\text{-cost}, L) / (i\text{-cost}, R - i\text{-cost}, L) \tag{1}$$

where:

- min-cost is the crisp value of the minimum cost path;
- i-cost is the crisp value of the path i;
- min-cost,R = min-cost$\cdot(1 + \alpha)$: i.e. the upper limit of the TFN associated to the minimum cost path;
- min-cost,L = min-cost$\cdot(1 - \alpha)$: i.e. the lower limit of the TFN associated to the minimum cost path;
- i-cost,R = i-cost$\cdot(1 + \alpha)$: i.e. the upper limit of the TFN associated to the path i;
- i-cost,L = i-cost$\cdot(1 - \alpha)$: i.e. the lower limit of the TFN associated to the path i;

The coefficient of preference for path i is given by the ratio between the shared part of the basis of the two TFNs and the base of the TFN associated to path i. The value of $C_{pref,i}$ for every path is between zero and one. In the Fig. 6 we see an example: $\alpha = 0.200$; min-cost = 3; min-cost,L = 2.4; min-cost,R = 3.6; i-cost = 4; i-cost, L = 3.2; i-cost,R = 4.8.

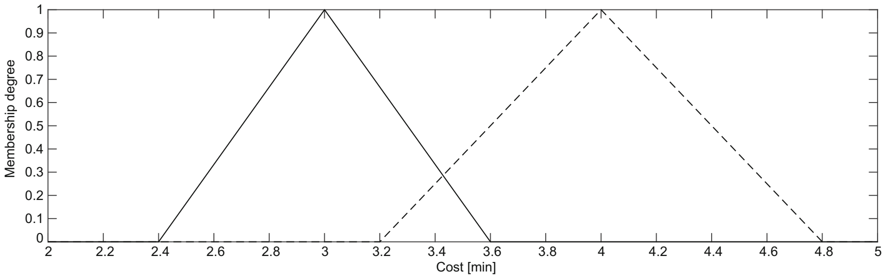

Fig. 6. Representation of path cost membership functions

The coefficient of preference is equal to 1 for the minimum cost path. For the other paths, it is lower as much as higher its crisp cost is. We assume that the probability of choice of a certain path (and consequently the demand fraction that will be assigned to it) is proportional to its coefficient of preference. We calculate the choice probability of the non-dominated paths by the normalisation of the coefficients of preference as follows:

$$p'_i = C_{\text{pref,i}} / \sum C_{\text{pref,i}} \tag{2}$$

Let us focus on the following explaining examples.

Let consider a user whose choice set is made up of two independent paths, namely A and B. Let the cost of B be 10 min greater than the cost of A. What path does the user choose? Under the assumption of *rational decision-maker* user, if his estimation of the path costs were precise, he/she would choose path A that maximize his/her utility. This is true for any value of the cost of paths A and B when B cost is greater than A cost. Therefore, the choice probability of A will be always equal to 100%. In the reality, users do not assume an exact estimation of the path costs [15] because of many source of uncertainty such as approximate reasoning of human beings, different tastes and perception of transportation supply etc. Therefore, choice probabilities depend (differently from the previous case) on the values of path costs and the degree of imprecision/approximation.

For example: let A cost be equal to 2 min and let B cost be equal to 12 min. The user knows that path A will cost *about 2 min* and path B will cost *about 12 min*. It is proper to assume that the choice set will be formed by the A path only. In other words, we expect that the choice probability of A will be equal to 100%. Now suppose that A and B costs are equal respectively to 40 min and 50 min. The user knows that path A will cost *about 40 min* and path B will cost *about 50 min*. Probably he/she will choice also path B, even if path A is clearly more convenient. As a result, we expect that the choice probability of path B will be greater than zero, even if it will be quite smaller than the choice probability of path A. Now suppose that A cost and B cost are equal respectively to 5 h and 20 min and 5 h and 30 min. The user knows that path A will cost *about 5 h 20'* and path B will cost *about 5 h 30'*. Probably the two paths look convenient in the same measure. As a result, the choice probability of the two paths will be very near to 50%.

In the following Table 1 we have summarized the choice probability values obtained by applying the proposed method in the described three examples.

Table 1. Choice probability values (α = 0.150)

Path	A	B	A	B	A	B
Cost	2′	12′	40′	50′	5h 20′	5h 30′
Choice probability	1.000	0.000	0.811	0.189	0.531	0.469

Note that the results are realistic for any combination of path cost values, and we obtained these results utilizing only one value of α for modelling the imprecision.

2.4 Overlapping Paths

In general, the paths belonging to the choice set could be partially overlapping [8]. If three or more paths form the choice set we may have unrealistic results (two of them may be overlapping in a certain measure). In this case, by applying the proposed method we may have unrealistic results. To avoid this problem it is necessary to properly define the estimation of choice probabilities.

We introduce an index for evaluating the overlapping degree of each path with the others. We describe the path overlapping in terms of length of shared sub-paths. Therefore, the index will be defined referring to the relevant arc lengths. The *coefficient of overlapping* for the path *i* is defined as follows:

$$C_{over,i} = \left[\left(L_{i,1}/L_i\right) + \left(L_{i,2}/L_i\right) + \ldots + \left(L_{i,i-1}/L_i\right) + \left(L_{i,i+1}/L_i\right) + \ldots + \left(L_{i,n}/L_i\right)\right] / \, n_{over} \tag{3}$$

with

- L_i is the length of the path *i;*
- $L_{i,1}(L_{i,2}, L_{i,3}\ldots)$ is the length of the shared sub-paths by the path *i* and the path 1 (2, 3,…,);
- n_{over} is the number of path partially overlapping in the choice set.

This coefficient is equal to zero for the independent paths. For the overlapping paths, it returns a value that increases proportionally to the length of the shared sub-paths. The value of the coefficient of overlapping is in every case less than one. Now we introduce the *coefficient of independence* as follows:

$$C_{ind,i} = 1 - C_{over,i}$$

It has the opposite meaning of the *coefficient of overlapping*. It is equal to 1 if path *i* is independent; it is less than 1 if path *i* has some shared sub-paths. In every case it is greater than zero. We can now calculate the choice probabilities of the non-dominated paths as follows:

$$p_i = C_{pref,i} \cdot C_{ind,i} / \sum C_{pref,i} \cdot C_{ind,i} \quad (4)$$

If the paths are all independent, all the *coefficients of independence* are equal to one. As a result, the choice probabilities are the same that we will obtain if we applied the normalisation of the coefficients of preference (as we describe in the previous section). Therefore, that case may be considered as a particular case. In this way, we obtain results more realistic in the case (very common in urban networks) of overlapping paths.

As example, let consider the following case. Let the choice set be made up of *n* paths: one independent path and (n−1) practically coincident paths. Let all the paths have the same cost. In this situation, all the coefficients of preference are equal to one. If we calculate the probability without consider the overlapping (i.e. by the Eq. (2)), we obtain that all the paths will have choice probability equal to 1/n. This is actually far from reality. In fact, because of the overlapping, users perceive only two effective alternatives, and will associate to them the same preference as they have the same costs. Therefore, the independent path (first alternative) will have probability of choice equal to 0.50, such as the totally of the remaining paths (second alternative). Each one of these paths has the same cost, so the choice probability of the generic overlapping path will be equal to 0.50/(n−1). If we apply the Eq. 4, we obtain exactly these results.

3 Numerical Analyses

Let us consider the network represented in Fig. 7 with six paths connecting the origin O with the destination D. Let the first one be independent and let its cost be equal to 40. Let the other paths cost be equal to 30. Because of the overlapping, paths 4, 5, 6 are practically the same.

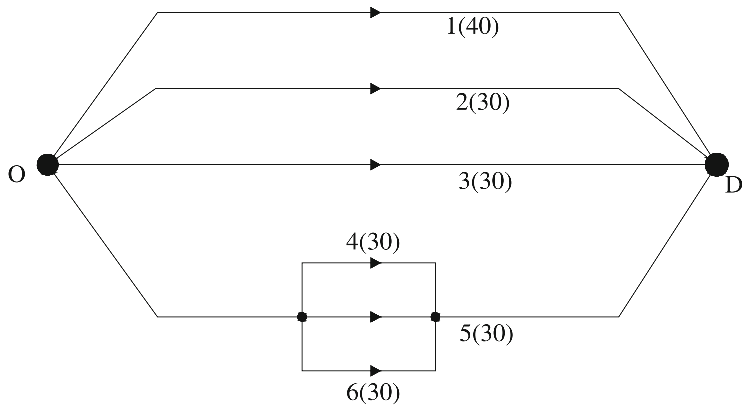

Fig. 7. Test network with overlapping paths

By applying the Eq. 3, the coefficient of overlapping is equal to zero for paths 1, 2, 3; and it is equal to 2/3 = 0,667 for paths 4, 5, 6. The expected path choice probability vector (P_1, P_2, P_3, P_4, P_5, P_6) must have the following characteristics:

1. $P_2 = P_3$ (because path 2 and path 3 have the same cost and are both independent);
5. $P_4 = P_5 = P_6$ (because path 4, path 5 and path 6 have the same cost and are overlapping in the same measure);
6. $P_1 < P_2$, P_3 (because all paths 1, 2, 3 are independent, but path 1 cost is greater than the others);
7. P_2, $P_3 = P_4 + P_5 + P_6$ (because paths 4, 5, 6 form only one alternative, due to their high overlapping, which is as convenient as the alternatives given by paths 2 and 3 - which are independent. Consequently, we expect that users associate the same choice probability to path 2, path 3 and the group of the paths 4, 5, 6).

The following Table 2 shows the results given by the proposed model where we assumed $\alpha = 0.150$. Estimated choice probabilities values reflects all the conditions just exposed, properly reproducing what we expect in route choice behaviour.

Table 2. Simulation results

Path	Lower limit	Crisp cost	Upper limit	$C_{pref,i}$	$\mathbf{P'_i}$	$C_{over,i}$	$C_{ind,i}$	$C_{pref,i} \cdot C_{ind,i}$	**Pi**
1	34.00	40.00	46.00	0.042	**0.008**	0.000	1.000	0.042	**0.012**
2	25.50	30.00	34.50	1.000	**0.198**	0.000	1.000	1.000	**0.329**
3	25.50	30.00	34.50	1.000	**0.198**	0.000	1.000	1.000	**0.329**
4	25.50	30.00	34.50	1.000	**0.198**	0.667	0.333	0.333	**0.110**
5	25.50	30.00	34.50	1.000	**0.198**	0.667	0.333	0.333	**0.110**
6	25.50	30.00	34.50	1.000	**0.198**	0.667	0.333	0.333	**0.110**

4 Equilibrium Assignment

In this case we considered the test network represented in (Fig. 8). To solve the traffic equilibrium problem the MSA (method of successive averages) has been used and then the equilibrium flows on the test network calculated. A sensitivity analysis has been carried out by considering different conditions of length or capacity of certain arcs and travel demand.

The considered network has six paths connecting the O/D pair form origin 1 to destination 9 (Fig. 8). The arc costs are calculated by the well known BPR cost function:

$$C = C_0 \cdot (1 + a \cdot (f_l / Cap_l)^{\gamma})$$

where:

- C_0 is the free flow cost;
- Cap_l is capacity of the arc l;
- f_l is the flow of the arc l;
- a and γ are parameters of the function.

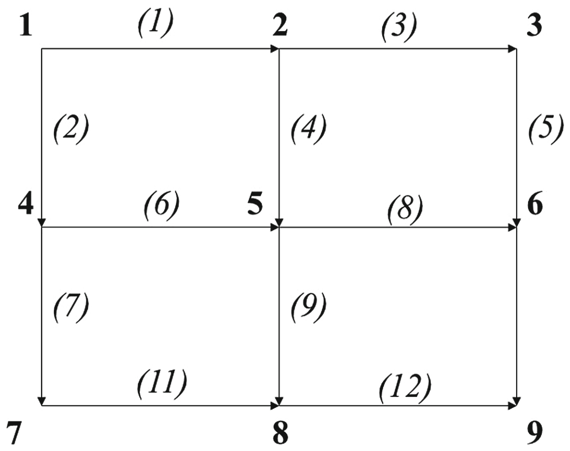

Fig. 8. Test network (Origin 1; Destination 9)

We assumed a = 2, $\gamma = 4$. We also assumed for each arc the capacity be 1000 and free-flow costs equal to 2. We perform several traffic assignments, considering different conditions of the network. For each condition, we applied our model achieving different tests for different values of α. We compared our results with those obtained by applying C-Logit model [7, 8], for different values of his parameters, θ and β_0.

The obtained results will be reported in the next section where the different considered conditions will be described.

4.1 Case 1 - Paths with the Same Free-Flow Cost

Let us consider the following situation: all the arcs have free-flow cost equal to 2, consequently free flow paths cost are all equal to 8. The assignment with C-Logit model [8] shows that the average user cost increases as much as the value of θ increases. The same happens applying the proposed model and increasing the value of α. In fact, θ and α represent (in different ways) the imprecision of the estimated path costs. If imprecision increases, we expect that more users will choice paths that are different from the most convenient one, with the consequence that the average user cost increases. This situation is correctly represented by increasing θ and α both in C-Logit and in our model.

This results were obtained for all the considered assignment cases, that are described as follows.

The vector of the arc flows at the equilibrium obtained by applying our model varies not much by changing the parameter α. We perform the traffic assignments by varying α in the interval 0.100 $\div$ 0.300. The index RMSE% is always lower than 5% between two probability vectors (obtained by our model with two different values of α, for the same configuration of the network).

Moreover, we saw good correspondence with C-Logit results assuming for the C-Logit parameters the values $\theta = 2.5$ and $\beta_0 = 1$.

Case 1a - Variation of the capacity of certain arcs. *Case 1a1* - Consider the arc 4 (from node 2 to node 5) with capacity equal to 500 (that is the half respect the previous case). We see that the average user cost increases respect to case "1" (we considered the comparison between the results obtained by one model without changing the values of its parameter). This is what we expect. In fact, the reduction of the capacity of one arc will induce an increasing of the network congestion (with the same demand), and obviously of the average user cost.

Case 1a2 - Consider now arc one with doubled capacity (thus equal to 2000). As we expect, we have from the results of the assignments that the average user cost is lower than in case "1". In fact, the increasing of the capacity of one arc will induce a reduction of the network congestion (with the same demand) and of the average user cost. The best correspondence for the results obtained by our model and C-Logit model are obtained when we assumed C-Logit parameter with these values: $\theta = 2.5$ and $\beta_0 = 1$.

Case 1b - Variation of travel demand. Consider the arcs having the same capacity and free flow costs as in case "1", but let the demand be equal to 1800. We see that the average user cost is higher than case "1", as we expect. In fact, the increasing of the demand will induce an increasing of the costs of the relative paths, so the total cost (and obviously the average user cost) will increase.

4.2 Case 2 – Variation of Free-Flow Cost

Case 2a - Let the free-flow cost of the arcs 3 and 5 be $C_0 = 1.5$ instead of 2. We see that the average user cost is lower than the case "1", as we expect. In fact, the reduction of the length of two arcs (and as a result the reduction of the correspondent free-flow cost) will generate a decreasing of the costs of the relative paths. Therefore, we see that the total cost and the average user costs decrease (with the same demand).

Case 2b - Consider the free-flow cost of the arc 4 and 8 very higher than the other arc costs. We expect that users will not consider these arcs, and consequently the relative paths (B [1-2-5-6-9], C [1-2-5-8-9], and D [1-4-5-6-9]).

The other paths (A [1-2-3-6-9], E [1-4-5-8-9], F [1-4-7-8-9]) have the same free flow cost. Moreover, path A is independent; path E and path F have the same degree of overlapping. We expect that the users will not perceive three different alternatives, but only two alternatives having the same length (the first one is path A and the second one is the group of paths E and F). The second alternative may be considered as a path containing a sub-path made of two parallel arcs having the same length. Practically, in this sub-path the capacity is doubled. Consequently, the congestion on them will be lower than on the other arcs. Thus, the second alternative is better than the first one (path A), so we expect that it will be chosen by the majority of the users. We also expect that on the overlapping sub-paths (E and F) the demand will be split in two equal proportions.

At the equilibrium, our model gives the following path choice probabilities (Table 3).

Table 3. Traffic assignment results (α = 0,200)

Path	Lower limit	Crisp cost	Upper limit	$C_{pref,i}$	$C_{over,i}$	$C_{ind,i}$	P_i
A [1-2-3-6-9]	6.840	8.549	10.259	1.000	0.000	1.000	0.430
B [1-2-5-6-9]	35.420	44.275	53.130	–	–	–	0.000
C [1-2-5-8-9]	21.268	26.584	31.901	–	–	–	0.000
D [1-4-5-6-9]	21.268	26.584	31.901	–	–	–	0.000
E [1-4-5-8-9]	7.115	8.894	10.673	0.884	0.250	0.750	0.285
F [1-4-7-8-9]	7.115	8.894	10.673	0.884	0.250	0.750	0.285

Paths B, C, D are dominated. In the Fig. 9, the representation of the TFNs relative to all the paths has been reported. Path E and path F have the same cost, such as path C and path D. As a result, the relative TFN are coincident.

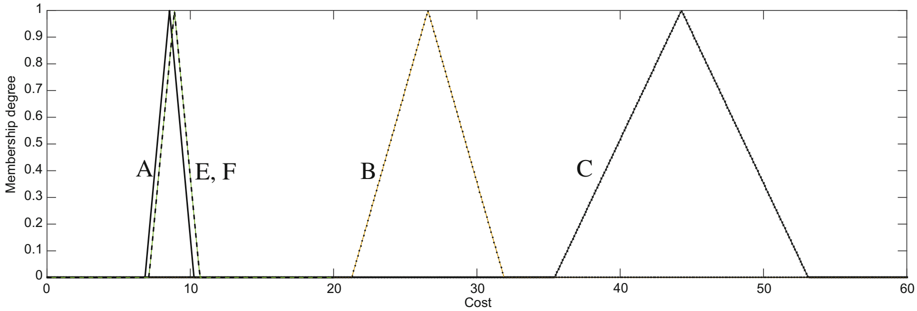

Fig. 9. Membership function

The model gives the expected results: the majority of the users chooses path E and path F, and fractions itself on them in equal parts. The vector of the choice probabilities changes not much by varying the value of α[3] (P_A = 0.439 if α = 0.100; P_A = 0.430 if α = 0.200; P_A = 0.425 if α = 0.300).

5 Conclusion

The aim of this work was to propose a path choice model where fuzzy numbers are utilised to represent the imprecision in the estimation of path costs in road network. The model allow to easily obtain the path choice probabilities overtaking some issue of Logit route choice model and explicitly considering users imprecision in cost perception. By using fuzzy numbers it is possible to consider the non-linearity between the value of path utility and the values of the attributes. The performed applications of the

[3] If α is greater than 0.520 path 3 and path 4 become non-dominated, if α is greater than 0.685 also path 2 become non-dominated (these values of α are not much representatives).

model has given realistic results in all the cases we have examined. It is important to remark that the proposed model is based on one parameter only. Moreover, we need only one value of this parameter to represent correctly users imprecision; both in first phase (selection of non-dominated paths) and in second phase (attribution of choice probabilities). Actually, users may have the problem of choosing between partially overlapping paths. Our model may give unrealistic results due to the IIA property as for Logit based models. To avoid this problem, we propose a correction of the calculation of choice probabilities introducing an index that allows to consider path overlapping. In this way, we obtained realistic results in the load assignment of the network in all the cases we have examined. In the traffic equilibrium assignment, we observe that parameter α is capable to represent the imprecision of the users when they estimate path costs. If the imprecision will increase then α increases. The results of the traffic assignment considering different conditions of the network show that there is a similarity with respect to the C-Logit model that we used as benchmark. The more is the imprecision we want to model, the higher the values of α and θ (for the C-Logit) must be. In this situation, we observe that the average user cost increases applying both our model and C-Logit model, and this is what we expect. Further investigations will be devoted to the analysis of real sized networks.

References

1. Akiyama, T., Kawahara, T.: Traffic assignment model with fuzzy travel time information. In: 9th Mini EURO Conference on Fuzzy Sets in Traffic and Transport Systems, Budva, Yugoslavia, September 1997
2. Arslan, T., Khisty, J.: A rational approach to handling fuzzy perceptions in route choice. Eur. J. Oper. Res. **168**(2), 571–583 (2006)
3. Ban, J., Liu, H., Yang, F., Ran, B.: A traffic assignment model with fuzzy travel time perceptions. In: Proceedings of the 12th World Congress on Intelligent Transport Systems, San Francisco, CA, USA (2005)
4. Bortolan, G., Degani, R.: A review of some methods for ranking fuzzy numbers. Fuzzy Sets Syst. **15**, 1–19 (1985)
5. Cantarella, G.E., Fedele, V.: Fuzzy utility theory for analysing discrete choice behaviour. In: Proceeding of the 4th International Symposium on Uncertainty Modelling and Analysis, pp. 148–154. IEEE Computer Society, Washington (2003)
6. Cantarella, G.E., Fedele, V.: Fuzzy utility models for discrete choice analysis. In: Proceedings of the 3rd International Conference on Traffic and Transport Psychology, Nottingham, UK, 5–9 September 2004
7. Cascetta, E.: Transportation System Analysis: Models and Applications. Springer, New York (2009)
8. Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, V.: A modified logit route choice model overcoming path overlapping problems. Specification and some calibration results for interurban networks. In: XIII International Symposium on Transportation and Traffic Theory, Lyon, France, pp. 697–711, 24 July 1996
9. Chang, M.S., Chen, H.K.: A fuzzy user-optimal route choice problem using a link-based fuzzy variational inequality formulation. Fuzzy Sets Syst. **114**, 339–345 (2000)

10. De Maio, M.L., Vitetta, A.: Route choice on road transport system: a fuzzy approach. J. Intell. Fuzzy Syst. **28**(5), 2015–2027 (2015)
11. Frejinger, E.: Route choice analysis: data, models, algorithms and applications (Doctoral dissertation, EcolePolytechniqueFederale de Lausanne) (2008)
12. Ghatee, M., Hashemi, S.M.: Traffic assignment model with fuzzy level of travel demand: an efficient algorithm based on quasi-Logit formulas. Eur. J. Oper. Res. **194**(2), 432–451 (2009)
13. Hawas, Y.E.: Development and calibration of route choice utility models: factorial experimental design approach. J. Transp. Eng. **130**(2), 159–170 (2004)
14. Henn, V.: Fuzzy route choice model for traffic assignment. Fuzzy Sets Syst. **116**, 77–101 (2000)
15. Henn, V., Ottomanelli, M.: Handling uncertainty in route choice models: from probabilistic to possibilistic approaches. Eur. J. Oper. Res. **175**, 1526–1538 (2006)
16. Kosko, B.: Fuzzy Thinking: The New Science Of Fuzzy Logic. Hyperion, New York (1993)
17. Marinelli, M., Caggiani, L., Ottomanelli, M.: Managing the uncertainty of data fusion from different sources in modelling route choice behaviour. In: Proceedings of the IEEE Conference on Intelligent Transportation Systems, ITSC, Las Palmas, Gran Canaria, pp. 202–207. IEEE Press (2015)
18. Murat, Y.S., Uludag, N.: Route choice modelling in urban transportation networks using fuzzy logic and logistic regression methods. J. Sci. Ind. Res. **67**(1), 19–27 (2008)
19. Okada, S., Soper, T.: A shortest path problem on a network with fuzzy arc lengths. Fuzzy Sets Syst. **109**, 129–140 (2000)
20. Ottomanelli, M., Dell'Orco, M., Sassanelli, D.: Modelling parking choice behaviour using Possibility Theory. Transp. Plann. Technol. **34**(7), 647–667 (2011)
21. Peeta, S., Yu, J.W.: A hybrid model for driver route choice incorporating en-route attributes and real-time information effects. Netw. Spat. Econ. **5**(1), 21–40 (2005)
22. Quattrone, A., Vitetta, A.: Random and fuzzy utility models for road route choice. Transp. Res. Part E **47**(6), 1126–1139 (2011)
23. Ramos, G.M., Daamen, W., Hoogendoorn, S.: A state-of-the-art review: developments in utility theory, prospect theory and regret theory to investigate travellers' behaviour in situations involving travel time uncertainty. Transp. Rev. **34**(1), 46–67 (2014)
24. Ridwan, M.: Fuzzy preference based traffic assignment problem. Transp. Res. Part C Emerg. Technol. **12**(3), 209–233 (2004)
25. Shao, C., Akiyama, T., Sasaki, T.: User equilibrium assignment on network with fuzzy route travel time. In: Proceedings of Sino-Japan Joint Meeting on Fuzzy Sets and Systems, Beijing, China, 15–18 October 1990
26. Teodorovic, D.: Fuzzy logic systems for transportation engineering: the state of the art. Transp. Res. Part A **33**, 337–364 (1999)
27. Teodorovic, D., Kikuchi, S.: Transportation route choice model using fuzzy inference technique. In: Proceedings of the First International Symposium on Uncertainty Modeling and Analysis, pp. 140–145. IEEE (1990)
28. Wang, F., Liao, H.L.: User equilibrium in traffic assignment problem with fuzzy N-A incidence matrix. Fuzzy Sets Syst. **107**, 245–253 (1999)